An Integrated Approach for Spare Parts Provisioning

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Abstract. This article addresses the problem of spare parts identification and provisioning for multi-component systems. A framework considering available technical, economical and strategic information is presented. Mathematical models are proposed to determine, for each spare part, the required quantity over a given planning horizon. The objective may be to maximize either the reliability or the availability of the system. Analytical models are proposed to determine the management parameters.

Key words: maintenance, spare parts, identification, inventory, optimization.

1. Introduction

When acquiring a system, one is often faced with the difficult question of identifying which of its components may fail to operate on the horizon considered and for which spare parts stocks are to be supplied. It should be noted that, contrary to standard parts, spare parts are designed for specific use and they can be acquired only from the manufacturer of the system or its authorized representatives. Their provisioning leadtime is generally long, if not often unknown. They cannot be resold easily. Consumption is typically governed by a random process. These parts are, additionally, subject to obsolescence and deterioration. Because they can be used to perform preventive replacement
(planned replacement) or corrective replacement (random replacement), classical mathematical models of inventory management [25] cannot be applied directly. Another dimension to consider stems from the fact that some parts may be remanufactured, or simply repackaged to be used as spares. Others can only be used once. First, we will deal with the latter category of parts and then we will tackle the first category in a subsequent section. Inventory management of spare parts is a strategic and economical issue for all organizations that operate equipment with operating characteristics deteriorating with age and use. Interruption of service can have high economical consequences. The unavailability of a component is certainly very detrimental both in terms of cost and continuity of production or service [30]. On the other hand, maintaining a high stock of spare parts can be expensive.

Section 2 of this article presents several decision tools for the identification of parts to be kept in stock. Section 3 deals with the determination of the spare parts quantity required to achieve a pre-determined performance for non-repairable systems. Section 4 proposes a mathematical model to find the quantity of repairable spare parts required to maintain a certain level of service. A conclusion of this work is presented in section 5.

2. Spare Parts Identification

In the absence of data and information to assess the degradation of a component and to estimate the probability of failure over the economic life of the system, we rely generally on the manufacturer’s recommendations. Manufacturers have often provided a list of components to keep in stock based on their own feedback, on data from accelerated testing conducted according to standard procedures or from more sophisticated analysis of the failure modes of the main components. To stay in business, equipment manufacturers are expected not only to meet the needs identified by the client, but also to anticipate these needs and to demonstrate that the products and services offered are equivalent if not superior to what is available on the market. Nowadays, customers may claim information that could help them gain maximum advantage from the system being acquired. More and more customers require their suppliers to provide them with lifetime and degradation information, analysis of failure modes, effects and criticality (FMEA), etc. Having access to these data, it becomes possible to identify components for which spare parts are to be kept. Thus, if the lifetime density function $f_i(.)$ or the lifetime cumulative distribution function $F_i(.)$ or the failure rate $h_i(.)$ or the reliability function $R_i(.)$ of component $i$ is known, then spare parts are to be held if

$$F_i(t) > F^*$$

where $F^*$ is the maximum failure risk the buyer (customer) is willing to accept during the mission duration of the system being bought.

It is also possible to base the decision on the criticality index obtained from the FMEA/FMECA of the system or by referring to similar equipment or on recommendations from internal or external experts. Figure 1 presents a generic process for identifying components for which spare parts are expected. For each equipment under consideration, all available data on lifetimes, repair times, suppliers, lead-times, etc., are gathered. If partial or no information is available, then the
decisions are based on the manufacturers’ recommendations or on information from owners of similar equipment. When complete information is available, then the decision-maker chooses the
criteria according to which the components will be evaluated to determine if they should be on the list of spare parts. A list of potential criteria is given in figure 1. According to the criteria retained, evaluation and final decision-making methods are to be selected next. Each component of the equipment is then evaluated according to the criteria and a total score is obtained. A list of potential methods is also provided in figure 1. A ranking of the components based on the final scores gives an ordered list of potential spare parts. Once the preliminary list of spare parts is established, it is subjected to filters to select the parts to hold in stock and those to be supplied as needed. An example of decision tree with filters is depicted in figure 2. This tree takes into account the cost of acquisition or production, repair costs, delays, whether there are early signs of failure or not, and if the component is a standard part or not. A standard part is a generic mass-produced part readily available at reasonable to low cost (e.g., seals, nuts and bolts, high replacement rate parts). Selected components are then ranked in order of importance. This classification allows to pay more attention to the components considered as being more important, especially if the list of spare parts includes a large number of components and the resources available are limited or scarce. Decision criteria most often used to justify that a spare part must be kept in stock to serve as a replacement are: criticality, reliability, availability, impacts of failure, failure rate and maintenance costs incurred in case of failure. In most cases, for simplicity or ignorance of the analysis tools available, only one or two of these criteria are considered in the analysis. For many organizations, the total cost of maintenance criterion is often the one used in the decision process. For each component \( i \), we calculate the critical ratio \( RG_i \) by

\[
RG_i = \frac{\text{Indirect Costs}}{\text{Direct Costs}}
\]

For more details on the direct and indirect costs, please refer to reference [9]. Any component \( i \) with \( RG_i \) ratio greater than 1, is then kept in store as a spare.

The analysis of failure modes, their effects and criticality, is increasingly used in industry to ensure sustainable use of assets. Each component is associated with a criticality index \( C \) obtained by multiplying the severity index \( S \), the probability of occurrence of the adverse event \( O \) and the difficulty of detecting it \( D \) (see [12, 6, 36])

\[
C = S \times O \times D
\]

For a component subject to several failure modes, a criticality index \( C_k \) is defined for each failure mode \( k \) (see [16]):

\[
C_k = K_A \cdot K_E \cdot \alpha_{kp} \cdot \beta_k \cdot \lambda_p \cdot T
\]

where

A system or component whose criticality index exceeds a predetermined threshold will be included in the list of potential spare parts.

In practice, the decision to store a part or not, may involve several criteria (costs, reliability, frequency of failure, response time, etc.) In the event that more than three criteria must be considered, multicriteria decision-making methods (MCDM) or tools have to be used. Several studies
$C_k$: criticality index for failure mode $k$

$K_A$: failure rate adjustment factor to compensate for actual operating conditions

$K_E$: failure rate adjustment factor to compensate for actual environmental conditions

$\alpha_{kp}$: proportion of failures of component $p$ due to failure mode $k$

$\beta_k$: conditional probability that failure mode $k$ will cause the failure

$\lambda_p$: failure rate of component $p$

$T$: mission length

Published in the literature have successfully used multicriteria methods for the identification and classification of equipment and spare parts (for example see [18] and [8]). The same tools can be adopted to generate a list of components to include in the priority list of potential replacement parts. Braglia et al. [7] used the AHP multicriteria classification method to classify the parts according to the impact of the failure, utilization, inventory problems and characteristics of parts. Schärlig [32] [31], Roy and Bouyssou [29] propose extensive reviews of multicriteria methods. Many computer programs and web sites offer the possibility of carrying out a multicriteria classification. Hammami [22] presents a very detailed review of computer programs dealing with multicriteria decision support. Eisenhawer et al. [17] propose a method of approximate reasoning based on fuzzy logic to establish a priority list of items to keep in stock for a nuclear facility. Once the components to be considered as spare (replacement) parts are identified, one has to determine the required quantities to be acquired during a given time period in order to achieve the expected performance levels. The following section will discuss models and methods for calculating the quantities of parts required during the economic life cycle of the system.

3. Determination of the required quantity of non-repairable spares

For each component requiring spare parts, it is important to estimate the required amount of spares needed throughout the economic life cycle of the equipment. To achieve this, one must estimate the average number of replacements at failure and, where applicable, the average number of preventive replacements. In this study, the data and information available will primarily be used to determine the density function $f(.)$, the distribution function $F(.)$, the survivor function or reliability $R(.)$, the failure rate $h(.)$ associated with the lifetime of the component under consideration. Figure 3 shows how field data is processed to obtain one of the four reliability characteristics. Knowledge of any one of these four characteristics is sufficient to obtain the other three. Table 1 recalls the relationships between these four functions.

For a component with lifetime density function $f(t)$ and negligible replacement duration, the average number of replacements at failure $M(t)$, with replacements carried-out with new spare
Figure 3: Field data processing diagram.

Table 1: Basic reliability relationships.

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(t)$</th>
<th>$R(t)$</th>
<th>$h(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>$-\int_0^t f(x)dx$</td>
<td>$\int_t^\infty f(x)dx$</td>
<td>$\frac{f(t)}{\int_t^\infty f(x)dx}$</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>$\frac{dF(t)}{dt}$</td>
<td>$-\int_t^\infty f(x)dx$</td>
<td>$1 - F(t)$</td>
</tr>
<tr>
<td>$R(t)$</td>
<td>$\frac{-dR(t)}{dt}$</td>
<td>$1 - R(t)$</td>
<td>$-\int_0^t R(t)dt$</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>$h(t)e^{-\int_0^t b(x)dx}$</td>
<td>$1 - e^{-\int_0^t b(x)dx}$</td>
<td>$-\int_0^t h(x)dx$</td>
</tr>
</tbody>
</table>

parts during a mission of length $t$, satisfies the following fundamental renewal equation:

$$M(t) = F(t) + \int_0^t M(t-x)f(x)dx.$$  

If $F^{(i)}(t)$ denotes the $i$–fold convolution of $F(t)$ with itself, then

$$M(t) = \sum_{i=1}^{\infty} F^{(i)}(t).$$

If at failure the component is minimally repaired without affecting its failure rate $h(t)$, then the average number of failure during the time interval $[0,t]$ is given by

$$M(t) = \int_0^t h(x)dx.$$
When the repair or replacement durations are random, the average number of failures during the time interval \([0,t]\) is given by

\[
M(t) = \sum_{i=1}^{\infty} G^{(i)}(t)
\]

where \(G^{(i)}(t)\) denotes the \(i\)-fold convolution of \(G(t)\) with itself, \(g(t) = \frac{dG(t)}{dt}\) being the convolution of the lifetime density function \(f(t)\) with the repair or replacement duration density function \(v(t)\) such that

\[
g(t) = \int_{0}^{t} f(t-x)v(x)dx.
\]

Closed-form expressions for the renewal function \(M(t)\) are only known to a relatively short list of distributions used in reliability and maintenance modeling, such as the Uniform, Exponential and Erlang distributions. However, several numerical methods have been proposed to compute \(M(t)\) (see [2, 24, 10]). Diallo and A¨ıt-Kadi [15] have proposed an approximation based on the Dirac function to compute \(g(t)\) and \(M(t)\) when repair or replacement durations are not negligible. Once \(M(t)\) is known, the average number \(n\) of spare parts required for the time interval \([0,t]\) is obtained by rounding its value to the next integer:

\[
n = \lceil M(t) \rceil.
\]

For a component replaced at failure and after \(T\) units of time, according to the age replacement policy (ARP), the upper bound of the expected number of spare parts \(n_a\) for a mission of length \(t\) is given by [5]:

\[
n_a = \left\lceil \frac{t \cdot [1 - R(T)]}{\int_{0}^{T} R(x)dx} \right\rceil.
\]

If the component is replaced at failure or at predetermined instants \(kT\) \((k = 1, 2, 3, \ldots)\) regardless of its age and state, according to the block replacement policy (BRP), then the expected number of spare parts \(n_b\) for a mission of length \(t\) is given by [5]:

\[
n_b = \lceil k [M(T) + 1] + M(t - kT) \rceil
\]

with \(kT \leq t < (k + 1)T\).

For some applications, a spare part can be considered as a stand-by component in the reliability point of view. The determination of the system reliability \(R_S(t, n)\) for a stand-by structure with \(n\) components allows to calculate the number of spares \(n\) to keep in stock to achieve a desired predetermined reliability level \(R^*\) for a given mission duration \(t\). This is equivalent to finding the smallest integer \(n^*\) such that:

\[
R_S [t, n^*(t)] \geq R^*
\]

\[
\int_{t}^{\infty} f^{[n^*(t)+1]}(x)dx \geq R^*
\]
where \( f^{(i)}(t) \) denotes the \( i \)-fold convolution of \( f(t) \) with itself. Once \( R_S(t, n) \) is known the number of spare parts to keep can be computed using a simple iterative algorithm from Aït-Kadi et al. [1] and depicted by figure 4.

Table 2 in Diallo et al. [13] gives the expressions of \( R_S[t, n^*(t)] \) for different configurations of the problem. In the general case, when \( k \) components are in operation and \((n - k)\) are kept in stock, the expression of \( R_S[t, n^*(t)] \) becomes:

\[
R_S[t, n^*(t)] = R_S[t, n^*(t), k]
\]

where

\[
R_S[t, n^*(t), k] = e^{-k\lambda t} \sum_{j=0}^{n-k} \frac{(k\lambda t)^j}{j!}.
\]

Note that the number of spare components can be calculated using a predetermined availability value \( A^* \). The approach is to find \( n^*(t) \) such that \( A[t, n^*(t)] \geq A^* \), where \( A[t, n^*(t)] \) is the availability of a stand-by structure consisting of \( n^*(t) \) reserve components and one component in operation. Diallo et al. [14] proposed a mathematical model for the maximization of the system’s availability under joint preventive maintenance and spare parts provisioning strategy.

4. Determination of the required quantity of repairable spares

A failed component, according to its degradation state, is repaired and put back in a “as new condition” or put in a state where it can resume operation. The acquisition cost of these repairable components is generally high. If the repair is found to be not feasible for technical, economical or other reasons, the failed component is sent to a recycling or disposal facility. The method based on the stand-by structure presented in the previous section can also be used here to determine the number of repairable components to use in a single system.

For a repairable component (as good as new) with failure rate \( h(t) = \lambda \) and repair rate \( \mu(t) = \mu \), the expression of \( R_S[t, n^*(t)] \) is given by

\[
R_S[t, n^*(t)] = e^{-zt}
\]
where

\[
z = \frac{(1 - \gamma)^2 \gamma^{n^*(t)} \lambda}{1 - \gamma^{n^*(t) + 1} \{1 + [n^*(t) + 1] (1 - \gamma)\}}
\]

and \(\gamma = \frac{\lambda}{\mu}\).

Inventory management of a fleet of repairable systems is much more complex than that of standard nonrepairable components. This problem occurs mainly in mining facilities, fleet maintenance, oil industry, civil aviation, military nuclear industry, etc. The complexity of the problem is usually due to the randomness of the failures, the varying restoration times, the uncertainties about the state of degradation of the component. Several analytical models, dealing with some variants of the problem of inventory management for repairable components, were published in the literature. Figure 5 presents, schematically, a configuration with two bases (operations centers) and a central depot for the maintenance of components and systems. The objective is to determine systems and components stock levels, at the bases and at the depot, in order to guarantee a given level of service at minimum cost. Since the repairs take time and given that any non-availability of the systems results in high costs, an inventory of systems in operational state is maintained at each base. Once a failure occurs at a base, a replacement system is taken from the stock and is immediately put in place to ensure continuity of service. The failed system is either repaired on site (at the base) with a given probability or sent to the repair center at the depot with a certain shipment leadtime. According to the level of degradation of the components, the repair can be undertaken or not. If the repair is not possible, a new one is ordered. When the repair is possible, spare components are used to carry-out the replacement. If replacement components are not available, an order is immediately placed. After repair, the system is sent back to the originating base (decentralized management) or kept in stock at the depot (centralized management). The repair time depends on the availability of spare parts, the repair capacities and the workload in the repair shops.

For the problem illustrated above, consider that the fleet has \(N\) machines (systems) in operation. These systems are independent and identically distributed each with a failure rate \(\lambda(t)\) and a repair rate \(\mu(t)\). A stock of \(y\) spare systems is held. A repair shop, consisting of \(c\) parallel channels, is used to repair systems that fail during operation. The production of goods or services is stopped whenever a total of \((N + y)\) systems have failed. Taylor and Jackson [35] were the first to apply queueing theory to solve this spare parts provisioning problem. Several studies have subsequently been devoted to the subject. These include, among others, the work done by Sherbrooke [33, 34] and others such as [4, 23, 27, 28, 3, 11, 26]. Two cases are distinguished. In the first case, the capacity of repair stations are assumed sufficient and therefore no waiting line is formed (see Sherbrooke [33, 34]). The modeling assumptions adopted lead to lower stocks than would be required to achieve the specified level of service [3]. In the second case, finite repair capacity is assumed. The analytical treatment becomes more complex as shown in [27, 3, 19]. A comprehensive review of articles on inventory management of repairable systems is carried out by Guide and Srivastava [21]. We will now present the model proposed by Gross et al. [19] to address the problem illustrated in figure 5. This is a Markovian model for a single echelon repairman problem whose transition diagram is shown in figure 6.

The number \(y\) of spare parts to keep in stock should allow to reach the service level \(NS\) defined
as the probability of having at least one spare machine in stock.

$P_i$ is the steady-state probability of having $i$ failed machines awaiting repair or being repaired. When the failure and repair rates are constant such that $\lambda(t) = \lambda$ and $\mu(t) = \mu$, then the $P_i$ are given by Gross et al. [19]:

$$P_i = \begin{cases} 
\frac{N^i}{i!} \left( \frac{\lambda}{\mu} \right)^i P_0 & \text{for } i = 1, \ldots, y \\
\frac{N^y N!}{(N - i + y)!} \left( \frac{\lambda}{\mu} \right)^i P_0 & \text{for } i = y + 1, \ldots, c - 1 \\
\frac{N^y N!}{(N - i + y)!c^i - c} \left( \frac{\lambda}{\mu} \right)^i P_0 & \text{for } i = c, \ldots, y + N
\end{cases}$$

for $c > y$
The expression of $P_0$ is given by

$$P_0 = \begin{cases} 1 + \sum_{i=1}^{c-1} \frac{N^i}{\mu^i} \left( \frac{\lambda}{\mu} \right)^i P_0 + \sum_{i=c}^{y} \frac{N^i}{(N-c)!(c-i)!(i-c)!} \left( \frac{\lambda}{\mu} \right)^i P_0 \for i = c, \ldots, y \end{cases}$$

for $c \leq y$.

The source population (the fleet of machines+spares) is usually finite $(N + y)$, therefore the probability that a failure is about to occur when there are $i$ broken machines in the repair shop is given by $Q_i$ (see Gross et al. [19]):

$$Q_i = \begin{cases} \frac{N \cdot P_i}{N - \sum_{i=0}^{y+N} (i-y) P_i} \for i = 0, \ldots, y-1 \end{cases}$$

for $i = 0, \ldots, y-1$.

Once the $Q_i$ are known, it suffices to find the smallest integer $y$ such that

$$\sum_{i=0}^{y-1} Q_i \geq NS$$

Logistical delays are not taken into account by this model. Moreover, the assumption that failure and repair rates are constant, sometimes causes decision-makers to question the validity of the results obtained by this model. Gross [20] studied the sensitivity of the model to the exponentiality assumption and derived easy rules of thumb to estimate the error induced by such a hypothesis. Kim et al. [27] present an algorithm to determine the level of the stock of spare parts required in the different bases in a two-tier system with a central depot. Note that in their model, the depot does not have a stock of spare parts. It only carries out repairs. The authors propose a mathematical model to minimize the average total cost of inventory management while satisfying a minimum service level. They also use the results of queueing theory to determine the probability of shortages and the likelihood of excess inventory. Logistical delays are included in their model.
5. Conclusion

An integrated approach for the identification and management of spare parts has been proposed. We have described a methodology for the identification of components for which replacement parts must be kept. For each spare part, analytical models are presented for determining the quantities required over a given operating horizon. Models of inventory management were then proposed for repairable systems. Several factors that affect system performance such that the replenishment leadtime and random demand are taken into account in the mathematical models presented. Because the machines and their operating environment tend to change over time, it is wise to frequently update the management parameters and decision variables to take into account any technological, economical and strategic change. It should also be worthwhile to implement proper maintenance procedures and monitoring of spare parts when they are stored for long periods of time by an appropriate control of their environment (moisture control, greasing, repositioning). Future research work should focus on the determination of inventory levels at bases and depot levels when risk-pooling initiatives are possible and lateral transshipments allowed. Setting of inventory levels of repairable spare parts also called “rotables” in some industries will be studied when scheduled inspections create random and deterministic demand on top of the demand generated by random failures.

Acknowledgements

The authors would like to thank the anonymous referees and the editors for their useful suggestions.

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